



**PANJAB UNIVERSITY**  
OF SCIENCE AND TECHNOLOGY

**FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

<b>QUALIFICATION:</b> Bachelor of science Honours in Applied Mathematics	
<b>QUALIFICATION CODE:</b> 35BAMS	<b>LEVEL:</b> 8
<b>COURSE CODE:</b> PDE 801S	<b>COURSE NAME:</b> PARTIAL DIFFERENTIAL EQUATIONS
<b>SESSION:</b> JUNE 2022	<b>PAPER:</b> THEORY
<b>DURATION:</b> 3 HOURS	<b>MARKS:</b> 90

<b>FIRST OPPORTUNITY EXAMINATION QUESTION PAPER</b>	
<b>EXAMINER</b>	Prof A. S. EEGUNJOBI
<b>MODERATOR:</b>	Prof O.D. MAKINDE

<b>INSTRUCTIONS</b>
<ol style="list-style-type: none"><li>1. Answer ALL the questions in the booklet provided.</li><li>2. Show clearly all the steps used in the calculations.</li><li>3. All written work must be done in blue or black ink and sketches must be done in pencil.</li></ol>

**PERMISSIBLE MATERIALS**

1. Non-programmable calculator without a cover.

**THIS QUESTION PAPER CONSISTS OF 3 PAGES** (Including this front page)

## QUESTION 1 [25 marks]

1. (a) From the following equations, form partial differential equations by eliminating the arbitrary contacts  $g, h$  and  $j$ .

i.

$$z = gxe^y + \frac{g^2e^{2y}}{2} + h \quad (5)$$

ii.

$$z = g(x + y) + h(x - y) + ght + j \quad (7)$$

- (b) By eliminating arbitrary functions from the followings, form the partial differential equation

i.

$$z = (x - y)f(x^2 + y^2) \quad (7)$$

ii.

$$f(x^2 - y^2, xyz) = 0. \quad (6)$$

## QUESTION 2 [20 marks]

2. Solve the following first order PDE

$$(a) (y^2 + z^2) \frac{\partial z}{\partial x} - xy \frac{\partial z}{\partial y} + xz = 0 \quad (5)$$

$$(b) (z + x) \frac{\partial z}{\partial y} + (y + z) \frac{\partial z}{\partial x} - x - y = 0 \quad (5)$$

$$(c) x(y - z) \frac{\partial z}{\partial x} + y(z - x) \frac{\partial z}{\partial y} = z(x - y) \quad (5)$$

$$(d) (x^2 + 2y^2) \frac{\partial z}{\partial x} = xy \frac{\partial z}{\partial y} + xz \quad (5)$$

## QUESTION 3 [25 marks]

3. (a) Classify, reduce to normal form and hence solve

$$3u_{xx} + 10u_{xy} + 3u_{yy} = 0 \quad (9)$$

- (b) Classify, reduce to normal form and hence solve

$$u_{xx} + 2u_{xy} + u_{yy} = 0 \quad (9)$$

- (c) Classify and reduce to normal form

$$y^2u_{xx} + x^2u_{yy} = 0 \quad (7)$$

## QUESTION 4 [20 marks]

4. (a) Solve

$$\frac{1}{4} \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0,$$

given that  $u(0, y) = 12e^{-5y}$ .

(10)

(b) Find the solution of the Cauchy problem

$$u_{tt} - c^2 u_{xx} = 0, \quad x \in \mathbb{R}, \quad t > 0, \quad u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \quad x \in \mathbb{R}.$$

(10)

**End of Exam!**